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RESEARCH REPORT No. EM-129

Two Dimensional Green's Function for a Right Angled Wedge Under an Impedance Boundary Condition

SAMUEL N. KARP

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Abstract

The Green's function is determined for a right angled wedge, when a constant impedance boundary condition is imposed on one face, while, on the other face, either the function or its normal derivative is required to vanish. The solution is shown to be expressible by quadratures in terms of Green's functions for simpler problems. The latter are obtainable by separation of variables. The far field amplitude is shown a priori to be a simple algebraic combination of the known far field amplitudes of the simpler Green's functions. This algebraic combination contains a parameter which is fixed by the requirement that the far field amplitude be a regular function.

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I. Introduction

The purpose of this paper is to present an elementary explicit solution of a problem of diffraction theory which hitherto has been unsolved. Mathematically stated, the problem is to determine the two dimensional Green's function $U(r, \theta: r_o, \theta_o)$ of the reduced wave equation $\Delta U + k^2 U = 0$, for the angular domain $0 \leq \theta \leq 3\pi/2$, $0 < \theta_o < 3\pi/2$, when the so-called impedance (or mixed) boundary condition $\lambda_1 \partial U/\partial\theta = \lambda_2 r U$ is imposed at the boundary $\theta = 3\pi/2$. Here λ_1 and λ_2 are certain complex constants and r and θ are polar coordinates. The time dependence $e^{-i\omega t}$ is assumed. On the boundary $\theta = 0$ we require that either U or $\partial U/\partial\theta$ vanishes. Speaking in terms of physics, we are considering the field of a line source of electromagnetic or acoustic waves in the presence of a wedge, one of whose faces ($\theta = 0$) is perfectly reflecting, while the other face has a special coating which possesses a prescribed impedance. It will be noticed that if either λ_1 or λ_2 vanishes, the boundary conditions simplify to those encountered at a conducting surface. Such problems, i.e., those of diffraction by perfectly conducting wedges, were the subject of some well-known investigations made by Sommerfeld[1] MacDonald[2] and Reiche[3]. A recent thoroughgoing discussion is due to Oberhettinger[4]. Nowadays the formal solution of the wave equation under these unmixed boundary conditions is a matter of the routine application of the method of separation of variables in polar coordinates. The peculiarity of the present problem is that the impedance boundary condition we have described makes it impossible to apply this method directly. The problem would be separable if $\frac{\lambda_1}{\lambda_2}$ were proportional to the radial variable r[5].

The principal result of the present paper is that for a rectangular wedge, the Green's function for $\lambda_1 \neq 0$, $\lambda_2 \neq 0$ can be expressed in elementary fashion, by quadratures and differentiations, in terms of the Green's function of the

problem with $\lambda_1 = 0$ or $\lambda_2 = 0$. But in the present analysis we restrict ourselves to the case in which $\lambda_2/\lambda_1 = \lambda$ is a complex number lying in the second quadrant or on the imaginary axis (see Section 2). The meaning of this restriction is given in Section 2, while the extension to wedges of other angles is discussed in the concluding section.

The solution of the three-dimensional vector problem of the diffraction of a skew incident plane wave by the wedge described here has been given by Karal and Karp[12]. This impedance boundary condition makes it impossible to reduce the vector problem to two scalar problems in the case of skew incidence; hence the present solution cannot be employed in [12].

Recently a two-dimensional scalar plane wave solution was given by Senior[11] using the method of Peters[9] (See footnote** on page 3) for wedges of any angle whatever. That solution can be employed to solve the scalar (but not the vector) three-dimensional plane wave problem. Also, it is to be expected that Senior's plane wave solution can be extended to complex angles of incidence, and that then, by contour integration over such angles of incidence one could obtain an expression for the Green's functions for any wedge angle. But the present procedure, and the present form of the result, which were worked out before Senior's paper became available, continue to seem most simple and useful for the special geometries discussed here and in the concluding section of our paper.

The method employed in our analysis is based on the introduction of an auxiliary wave function $V = (\partial/\partial x - \lambda)U$. The operator $(\partial/\partial x - \lambda)$ commutes with both the wave equation and the boundary conditions. (In the discussion in this paragraph we are regarding U as a function of the cartesian coordinates $x = r\cos\theta$, $y = r\sin\theta$.) In terms of these coordinates, the impedance boundary condition for U becomes $[(\partial/\partial x)-\lambda]U(0-,y) = 0$ for $y < 0$. It therefore

follows from the definition of V that this new function obeys simple boundary conditions $V(x,0+) = 0$ or $\partial V(x,0+)/\partial y = 0$, for $x > 0$, and $V(0-,y) = 0$ for $y < 0$. The idea is to exploit the simplicity of the boundary conditions in order to determine V. Once V is obtained, the determination of U is reduced to the integration of the ordinary differential equation $\partial U/\partial x - \lambda U = V(x,y)$.*

This idea is taken from a paper on water waves on sloping beaches[6] wherein Stoker obtains a progressive wave solution of the equation $\Delta U - k^2 U = 0$ in the angular region $-\pi/2 \leq \theta \leq 0$, i.e., $(x \geq 0,\ y \leq 0)$. The boundary conditions are that $U_y - U = 0$ for $y = 0$, $x > 0$, and $U_x = 0$ for $x = 0$, $y < 0$. The operator, $\Big| = \partial/\partial x\,[(\partial/\partial y) - 1]$ is used, leading to a function $V = \partial/\partial x[(\partial/\partial y) - 1]U$ which obeys simple boundary conditions.** Although our problem refers to the angular region $0 < \theta < 3\pi/2$, rather than to Stoker's region, $-\pi/2 < \theta < 0$, the same method can be used to simplify the boundary conditions, since the boundaries of both domains have the same description in cartesian coordinates.

While the similarity of boundary conditions allows us to adapt the essential first step of Stoker's work to our purposes, we have to proceed quite differently thereafter. This is partly a result of the fact that our excitation is more complicated, but is more especially due to the essential differences between the problems of diffraction and water wave theory. One difference, for example, is that we require (See section 2) a solution which

---

* Actually we first determine V to within an additive multiple of a certain known function, and find a corresponding expression for U by integration. The unknown coefficient is then determined by reapplying the boundary condition for U.

** Stoker also indicates how one may treat angular spaces of extent $\pi/2n$, when $n=1,2\ldots$ More general angular spaces $p\,\pi/2n$ (p being odd and less than 2n) are treated by H. Lewy[7] for the case of $k=0$. The question of uniqueness has been discussed by A. Weinstein[8]. A complete treatment of this problem for all domains of extent $\leq \pi$ has been given by A. Peters[9] and M. Roseau[10].

is finite at the edge. Such a solution, even for plane wave excitation, is not trivial in the diffraction problem. On the other hand, in the corresponding water wave problem, where the angular domain is $-\pi/2 < \theta < 0$, it is easy to obtain a solution finite at the edge. The problem treated by Stoker involved the determination of a solution which becomes infinite at the edge in a manner which allows a special behavior at infinity.

We close the introductory section by outlining the subsequent developments. In Section 2 we give the detailed mathematical formulation of our problem, and also comment on the significance of the conditions we impose. The solution is determined in Section 3, in terms of elementary operations upon the solutions of simpler problems for which either λ_1 or λ_2 is zero. The latter solutions are of the type which are easily obtainable by separation of variables. Simple expressions are obtained for the far field in Section 4. In addition, we show that consideration of certain a priori properties of the far field leads to a simpler representation of a certain constant occuring in the solution. Section 5 is devoted to a brief discussion of the scope of the method, and to concluding remarks.

II. Formulation of the problem

We shall proceed to formulate the boundary value problem mathematically and then we shall discuss the significance of the formulation.

The problem we solve is defined by conditions (1) and (5) below. The notation is explained in Figure 1.

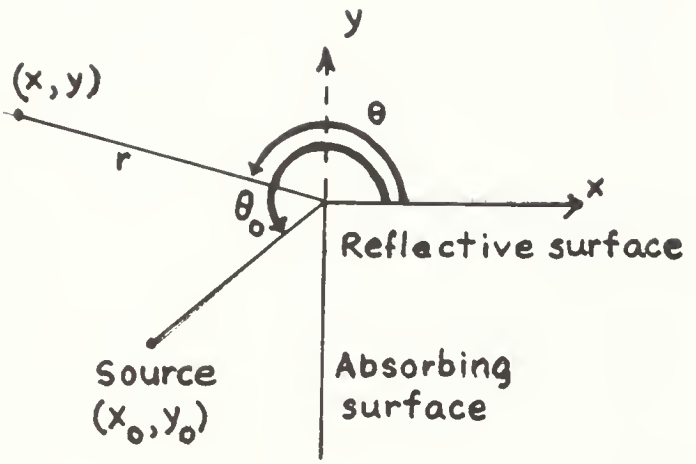

Figure 1

(1) $U_{xx} + U_{yy} + k^2U = \delta(x-x_o)\,\delta(y-y_o)$; $0 \leq r < \infty$; $0 < \theta < 3\pi/2$

(2a) $U(x,0+) = 0$, $x > 0$

or

(2b) $U_y(x,0+) = 0$; $x > 0$,

(3) $U_x(0-,y) - \lambda U(0-,y) = 0$; $y < 0$, $\mathrm{Re}\ \lambda \leq 0$, $\mathrm{Im}\ \lambda > 0$,

(4) $\lim_{r\to\infty} \sqrt{r}\,(U_r - ikU) \to 0$, $0 < \theta < 3\pi/2$

(5) U is finite except at the point (x_o,y_o) .

It is appropriate to comment on the meaning of the conditions we have enumerated. Equation (1) arises in a standard way in Electromagnetics or Acoustics. In Acoustics, condition (2a) represents a soft obstacle, while condition (2b) represents a rigid obstacle. In Electromagnetics, conditions (2a) and (2b) refer to a conducting wall, when (a) the electric field or (b), the magnetic field, is perpendicular to the xy plane, respectively. The first part of condition (3) is the impedance boundary condition which expresses, electromagnetically speaking, the existence of an enforced proportionality between the tangential components of the electric and magnetic fields at a

specially coated wall. In an acoustic context this condition expresses a proportionality of pressure and velocity, the effect being likewise caused by a special coating. The second part of condition (3) deals with the special character of the complex number λ. As will be illustrated shortly, the requirement $\mathrm{Im}\ \lambda > 0$ implies absorption of energy at the vertical part of the wedge surface. The condition that $\mathrm{Re}\ \lambda \leq 0$ is necessary for the validity of the procedure employed in the present paper, since certain integrals (see equation 11) would otherwise diverge. Physically speaking, this condition excludes the existence of surface waves on the vertical face of the wedge. By a modification of the present method one can obtain a solution valid for $\mathrm{Re}\ \lambda > 0$, and we plan to discuss this subject in a separate work dealing with the excitation of surface waves on wedges.

Condition (3) implies that one face of the wedge absorbs energy. There is a special case of condition (3), namely, when λ/ik is real and $0 < \lambda/ik \leq 1$, when the absorption is especially significant. We shall now explain this point. The value of the parameter λ/ik is an intrinsic property of a surface. Consider the effect of an infinite wall made of a material possessing the property under discussion, and placed along the y axis, on a plane wave incident from the left and traveling in direction β, where $\cos\beta > 0$. The incident wave is given by the expression $\exp[ik(x\cos\beta + y\sin\beta)]$, and the reflected wave is $[(ik\cos\beta - \lambda)/(ik\cos\beta + \lambda)]\exp[-ikx(\cos\beta + iy\sin\beta)]$. It is easily seen that the reflection coefficient vanishes for the particular value of $\cos\beta$ given by the formula $\cos\beta = \lambda/ik$. Thus the wall absorbs the excitation perfectly in this case. Now consider a more complicated problem. Let a plane wave be incident on the right angled wedge we have described earlier. This problem, which we denote by P, is a limiting case of the line source problem we actually treat in the present paper (since we have the same boundary conditions) because a plane wave is an idealization of a distant line

source. In view of our previous thought experiment with the reflection at an infinite wall, we can say that the solution of problem P represents the diffraction of a plane wave by a wedge, one of whose surfaces absorbs plane waves which are incident at the angle arc cos (λ/ik). Finally, therefore provided that $0 < \lambda/ik \leq 1$, the solution of the problem actually treated in the present paper represents the diffraction of the field of a line source, by a wedge which has one surface which perfectly absorbs a certain plane wave.

We now turn to an examination of the remaining conditions of our problem. These conditions frequently occur in diffraction theory. Condition (4) expresses Sommerfeld's radiation condition. In (5) we have a convenient form of edge condition which, combined with (4) will be seen to be sufficient to secure uniqueness of our solution. It is, incidentally, also sufficient for uniqueness in connection with the classical problem of a perfectly conducting wedge, as is readily apparent from the considerations which lead to a solution of that problem by separation of variables, using angular eigenfunctions.

III. Deduction of the solution

Having discussed the formulation of the boundary value problem, we turn to its solution. Our method of analysis begins with the introduction of a function

(6) $\quad V(x,y) = (\partial/\partial x - \lambda)\, U(x,y).$

If there exists a function U fulfilling the requirements (1) to (5), it follows that V must obey the conditions

(7) $\quad \triangle V + k^2 V = (\frac{\partial}{\partial x} - \lambda)\, \delta(x-x_o)\, \delta(y-y_o) = -(\frac{\partial}{\partial x_o} + \lambda)\, \delta(x-x_o)\, \delta(y-y_o)$

(8) $\quad V(0-,y) = 0 : \quad y < 0,$

and also conditions (2) and (4). The function V need not be finite at the origin, since it involves a differentiation of U. By applying the operator $-(\frac{\partial}{\partial x_o} + \lambda)$ to the unique Green's function fulfilling conditions (2), (4), (5) and

(8) we obtain a unique function Ψ, which satisfies not only the conditions imposed on V (namely (2), (4), (7) and (8)) but also condition (5). Then using separation of variables, and applying the radiation condition, it can be seen that the most general function obeying conditions (2), (4), (7) and (8) is obtained by adding to Ψ a singular solution of the form

$$(8a) \quad \sum_{n=0}^{\infty} H_{\nu_n}^{(1)}(kr)\left[\alpha_n \cos \nu_n\theta + \beta_n \sin \nu_n\theta\right]$$

where, if (2a) holds, then $\alpha_n = 0$, and $\nu_n = 2n/3$, while if (2b) holds, then $\beta_n = 0$, and $\nu_n = 2n+1/3$. However, since U must be finite, V can not be too singular. Hence, of all the terms in (8a), only the non-vanishing term of lowest index can be employed in the construction of V. Thus V contains just one parameter, which is as yet free.

We shall now show that the boundary value of U, or its normal derivative on the positive x axis determines the value of this parameter. In what follows we shall designate by a tilda the class of solutions in which the free parameter has not yet been fixed. The solution of the problem after we fix the parameter so that the boundary conditions are satisfied will be written without the tilda. Thus we designate by $\tilde{U}$ a wave function obeying the radiation condition and determined by $\frac{\partial}{\partial x}\tilde{U} - \lambda\tilde{U} = \tilde{V}$, where $\tilde{V}$ is any member of the one parameter class of functions satisfying conditions (2), (4), (7) and (8). If condition (2a) is imposed then $\tilde{V}(x,0) = 0$ on the positive x axis. Thus we have

$$(\frac{\partial}{\partial x} - \lambda)U = 0 ,$$

and consequently $U(x,0+)$ has the form $c \exp \lambda x$ for $x > 0$. c is a constant which depends in a complicated way on the value of the free parameter used in defining V. The vanishing of $\tilde{V}(x,0+)$ does not imply the vanishing of $U(x,0+)$; only if the free parameter is properly chosen will c be equal to zero. A similar argument can be made when condition (2b) is imposed.

We shall now represent $\tilde{V}$ in a new form in order to determine the function conveniently. Consider, to this end, the Green's functions G and N, defined by equation (1), (2), (4), and (5) and the respective boundary conditions.

(9) $\quad G(0-, y; x_o, y_o) = 0$

$\quad N_x(0-, y; x_o, y_o) = 0 \qquad\qquad$ for $y < 0$.

These functions G and N are obtainable by standard separation of variables techniques. In case (a), where equation (2a) is assumed, we have:

$$G = -\frac{2i}{3}\sum_{n=1}^{\infty} J_{\frac{2n}{3}}(k\rho_1)\, H^{(1)}_{\frac{2n}{3}}(k\rho_2)\, \sin\tfrac{2n}{3}\theta\, \sin\tfrac{2n}{3}\theta_o$$

(9a)

$$N = -\frac{2i}{3}\sum_{n=o}^{\infty} J_{\frac{2n+1}{3}}(k\rho_1)\, H^{(1)}_{\frac{2n+1}{3}}(k\rho_2)\, \sin\tfrac{2n+1}{3}\theta\, \sin\tfrac{2n+1}{3}\theta_o,$$

while in case (b) we have:

$$G = -\frac{2i}{3}\sum_{n=o}^{\infty} J_{\frac{2n+1}{3}}(k\rho_1)\, H_{\frac{2n+1}{3}}(k\rho_2)\, \cos\tfrac{2n+1}{3}\theta\, \cos\tfrac{2n+1}{3}\theta_o$$

(9b)

$$N = -\frac{i}{3}\sum_{n=o}^{\infty} \varepsilon_n\, J_{\frac{2n+1}{3}}(k\rho_1)\, H_{\frac{2n+1}{3}}(k\rho_2)\, \cos\tfrac{2n}{3}\theta\, \cos\tfrac{2n}{3}\theta_o\ .$$

Here $\rho_1 = \min(r,r_o)$, $\rho_2 = \max(r,r_o)$, $\varepsilon_o = 1$, $\varepsilon_1 = \varepsilon_2 = \ldots = 2$.
From equation (1) we infer that

$$(9c) \qquad (\triangle+k^2)\left[\frac{\partial G}{\partial x_o} + \frac{\partial N}{\partial x}\right] = \left(\frac{\partial}{\partial x_o} + \frac{\partial}{\partial x}\right)\delta(x-x_o)\,\delta(y-y_o) \equiv 0.$$

Hence the function $H \equiv \frac{\partial G}{\partial x_o} + \frac{\partial N}{\partial x}$ is a solution of the homogeneous wave equation. The definition of H, taken together with equations (9) and (9a) (or (9b)), implies that this function obeys conditions (2a) (or (2b)) and (4), vanishes at $x = 0$, $y < 0$, is radiating at infinity and behaves at the origin like the non-vanishing term of lowest index in the series we have used to represent the singular solution to be added to V (see equation 8a). [The proof of this

statement is straightfoward. First of all $\partial G/\partial x_o$ vanishes at $x = 0$, $y < 0$, as can be seen from (9), and this implies that H also vanishes there. Conditions (2) and (4) are fulfilled by G and N, hence by $\partial G/\partial x_o$ and $\partial N/\partial x$, and therefore also by H. At the origin the leading term of G, and of $\partial G/\partial x_o$ as well, is finite. The leading term of $\partial N/\partial x$ can be verified to have the behavior specified.] It follows that the singular solution, which must be added to Ψ in order to obtain the most general $\tilde{V}$, is a multiple of H. On the other hand, the function Ψ, since it is a solution of (7), satisfying (8), (2), (4), and (5), can be written in the form $\Psi = -(\partial/\partial x_o + \lambda)G$. Consequently, we obtain a sufficiently general representation of $\tilde{V}$ by writing:

$$(10) \qquad \tilde{V} \equiv (\tilde{U}_x - \lambda\tilde{U}) \equiv -(\partial/\partial x_o + \lambda)G + A(\partial G/\partial x_o + \partial N/\partial x).$$

The determination of the particular function V which leads, through solution of equation (6), to the function U characterized by (1) through (5), thus reduces to the proper selection of the constant A. Notice: when $\lambda \to 0$, the boundary conditions for $\tilde{U}$ become those for N, so that we must expect that the correct value for $A \equiv A(\lambda)$ then tends to unity, and then (10) will read $\tilde{V} \equiv \tilde{U}_x \equiv N_x$.

By means of equation (10) we have now represented the class of functions $\tilde{V}$, of which the desired function V is a member. But we have yet to determine the constant A, which distinguishes V from the other $\tilde{V}$. In order to determine A we first determine, for each $\tilde{V}$, the function $\tilde{U}$ described in the text following (8a). The function $\tilde{U}$ is a solution of the ordinary differential equation

$$(10a) \qquad \frac{\partial\tilde{U}}{\partial x} - \lambda\tilde{U} = \tilde{V},$$

and, in addition, it is a wave function which is outgoing at infinity and which has the desired singularity at $x=x_o$, $y=y_o$, while remaining finite at the origin. We can then impose conditions (2) on U. If we specialize A so that it validates condition (2) then $\tilde{U}$ will have been specialized to the function U required by (1) through (5).

Consider the function defined by

$$(11) \quad \tilde{U} = e^{\lambda x} \int_{-\infty}^{x} e^{-\lambda \xi} \tilde{V}(\xi, y) d\xi \ .$$

This is an appropriate function $\tilde{U}$ if the integral exists, if it satisfies the wave equation and if it obeys the outgoing wave condition.

The existence of the integral is guaranteed if we choose Re $\lambda < 0$. For Re $\lambda = 0$, the fact that $\tilde{V}$ satisfies the radiation condition assures convergence. Thus $\tilde{U}$ exists when Re $\lambda \leq 0$.

That the wave equation is satisfied can be shown from the fact that

$$\tilde{U}_x = \tilde{V} + \lambda \tilde{U} \ , \quad \text{and therefore}$$

$$\tilde{U}_{xx} = \tilde{V}_x + \lambda \tilde{U}_x = \tilde{V}_x + \lambda \tilde{V} + \lambda^2 U \ .$$

Also

$$\tilde{V}_{yy} = - \tilde{V}_{\xi\xi} - k^2 \tilde{V} - \left(\frac{\partial}{\partial x_o} + \lambda \right) \delta(\xi - x_o) \, \delta(y - y_o) \ .$$

By inserting (11) into the wave equation (1), using the above expressions for $\tilde{U}_{xx}$ and $\tilde{V}_{yy}$, then integrating twice by parts, we obtain

$$\tilde{U}_{xx} + \tilde{U}_{yy} + k^2 \tilde{U} = - \left(\frac{d}{dx_o} + \lambda \right) \int_{-\infty}^{x} e^{\lambda (x - x_o)} \, \delta(\xi - x_o) \, \delta(y - y_o) d\xi$$

$$= - \left(\frac{d}{dx_o} + \lambda \right) e^{\lambda (x - x_o)} \, h(x - x_o) \, \delta(y - y_o)$$

$$= - e^{\lambda (x - x_o)} \frac{\partial}{\partial x_o} h(x - x_o) \, \delta(y - y_o)$$

$$= e^{\lambda (x - x_o)} \, \delta(x - x_o) \, \delta(y - y_o)$$

$$= \delta(x - x_o) \, \delta(y - y_o) \ ,$$

where $h(x - x_o)$ is the unit step function. Thus $\tilde{U}$ defined by (11) satisfies the

wave equation.

To see that $\tilde{U}$ is outgoing at infinity, we rewrite equation (11) by introducing the new variable of integration $t = x - \xi$. We find:

$$(11a)\quad \tilde{U}(x,y) = \int_0^\infty e^{\lambda t}\tilde{V}(x-t,y)dt = \int_0^M e^{\lambda t}\tilde{V}(x-t,y)dt + \int_M^\infty e^{\lambda t}\tilde{V}(x-t,y)dt$$

where M is an arbitrarily large positive number. The function $\tilde{V}(x-t,y)$ represents the appropriate function $\tilde{V}$ for a wedge whose vertex is at the point (t,o) rather than at the origin.

In preparation for what follows we now think of $\tilde{V}$ as being decomposed into the effect of the source, and the effect of the wedge. If we let $\tilde{V}_{inc}$ denote the former, then we have

$$\tilde{V}_{inc} = -\left(\frac{\partial}{\partial x_o} + \lambda\right)\ H_o\left(k\sqrt{(x-x_o)^2 + y^2}\ \right).$$

Since this function has an appropriate singularity at (x_o,y_o), and is regular everywhere else, it represents the effect of the source. The function $\tilde{V}_{inc}$ is an outgoing function.

We now return to a consideration of equation (11a). If we choose M large enough, then we can ignore the diffraction effect in the second integral, so that $\tilde{V}(x-t,y) \to \tilde{V}_{inc}(x,y)$ for $t > M$. Then $\int_M^\infty e^{\lambda t}\tilde{V}(x-t,y)dt \to -\frac{V_{inc}}{\lambda}(x,y)e^{\lambda M}$. Hence, this term is outgoing. Since $\int_o^M e^{\lambda t}\ \tilde{V}(x-t,y)dt$ is a finite superposition of outgoing waves this integral,too, can easily be shown to be outgoing. Thus $\tilde{V}$ as defined by (11) is outgoing, as was to be shown. We shall now determine the arbitary constant in order to have a solution of our particular problem. To do this we apply conditions (2a) and (2b). Condition (2a) requires that

$$(12a)\quad \tilde{U}(x,0^+) = \lim_{y\to 0} e^{\lambda x}\int_{-\infty}^{x} e^{-\lambda\xi}\ V(\xi,y,A)d\xi = 0\ ,\ x > 0$$

while condition (2b) requires that

$$(12b)\qquad \tilde{U}_y(x,0^+) = \lim_{y\to 0} \frac{\partial e}{\partial y}^{\lambda x} \int_{-\infty}^{x} e^{-\lambda\xi}\tilde{V}(\xi,y,A)d\xi = 0, \quad x > 0.$$

In (12b) we cannot differentiate under the integral sign and then let $y = 0$, since $\tilde{V}_y(x,0)$ is too singular to be integrable. However if we insert the second equality of (10) into (11) and integrate by parts we obtain

$$(13)\qquad \tilde{U} = e^{\lambda x} \int_{-\infty}^{x} e^{-\lambda\xi} \left\{ \left[(A-1) \frac{\partial}{\partial x_o} - \lambda \right] G + \lambda AN \right\} d\xi + AN(x,y).$$

In this formula we can differentiate under the integral sign and let $y = 0$. For uniformity we shall use (13) in treating case (a) as well as case (b). Now using condition (2a), (2b), respectively, our requirement becomes:

$$(14a)\qquad \tilde{U}(x,0^+) = e^{\lambda x} \int_{-\infty}^{0} e^{-\lambda\xi} \left\{ \left[(A-1) \frac{\partial}{\partial x_o} - \lambda \right] G(\xi,0) + \lambda AN \right\} \cdot d\xi = 0$$

$$(14b)\qquad \tilde{U}_y(x,0^+) = e^{\lambda x} \int_{-\infty}^{0} e^{-\lambda\xi} \left\{ \left[(A-1) \frac{\partial}{\partial x_o} - \lambda \right] G_y(\xi,0) + \lambda AN_y(\xi,0) \right\} d\xi = 0 .$$

The integrals are from $-\infty$ to zero since positive values of ξ do not contribute to the value of the integral at $y = 0$ because of the boundary conditions obeyed by $G(\xi,y)$ and $N(\xi,y)$.

In case (a) we find

$$(14c)\qquad A = \frac{\displaystyle\int_{-\infty}^{0} e^{-\lambda\xi} \left[\frac{\partial G}{\partial x_o}(\xi,0;x_o,y_o) + \lambda G(\xi,0;x_o,y_o) \right] d\xi}{\displaystyle\int_{-\infty}^{0} e^{-\lambda\xi} \left[\frac{\partial G}{\partial x_o}(\xi,0;x_o,y_o) + \lambda N(\xi,0;x_o,y_o) \right] d\xi},$$

while in case (b) we find

$$(14d)\qquad A = \frac{\displaystyle\int_{-\infty}^{0} e^{-\lambda\xi} \left[\frac{\partial G_y}{\partial x_o}(\xi,0+;x_o,y) + \lambda G_y(\xi,0+;x_o,y_o) \right] d\xi}{\displaystyle\int_{-\infty}^{0} e^{-\lambda\xi} \left[\frac{\partial G_y}{\partial x_o}(\xi,0+;x_o,y_o) + \lambda N_y(\xi,0+;x_o,y_o) \right] d\xi} .$$

The functions G and N are known (see equations (9a) and (9b)). Hence A is determined. If we insert A into (10) and then (10) into (11), and remove the now superfluous tilde, we have an explicit but complicated expression for U in terms of the simpler functions G and N. A simpler formula will be given in the next section. It will be derived for the case $0 < \lambda/ik \leq 1$, and its justification for the general case will be briefly indicated.

IV. The far field

In the present section we determine the form of U at infinity, and also indicate another and simpler method for determining the constant A in the solution U, in certain cases, from far field considerations alone.

We begin by observing that U, G, and N all obey the radiation condition, whence we have the asymptotic formulas,

$$(15)\quad G \to \frac{eikr}{\sqrt{r}}\, g(\theta;\, x_o,y_o) \qquad N \to \frac{e^{ikr}}{\sqrt{r}}\, n(\theta;x_o,y_o)$$

$$U \to \frac{e^{ikr}}{\sqrt{r}}\, u(\theta;x_o,y_o) \qquad V \to \frac{e^{ikr}}{\sqrt{r}}\, v(\theta;x_o,y_o),$$

as $r \to \infty$ with a fixed value of θ in the interval $0 < \theta < 3\pi/2$. Here the functions g and n are regarded as known, but the function $u(\theta;x_o,y_o)$ is yet to be found. The function $V \equiv U_x - \lambda U$ is given in terms of G and N by (10). The coefficient A in (10) is given by equations (14c) or (14d). The function $v(\theta;x_o,y_o)$ is the far field amplitude corresponding to V. We apply equation (6), carry out the differentiations in polar coordinates, and collect terms of order $r^{-1/2}$. We thus obtain

$$(16)\quad (ik \cos\theta - \lambda)\, u(\theta) - v(\theta) = 0.$$

Hence we find

$$(17)\quad u(\theta;x_o,y_o) = \frac{[(A-1)\,\partial/\partial x_o - \lambda]\, g(\theta;x_o,y_o) + ik \cos\theta\, An(\theta;x_o,y_o)}{ik \cos\theta - \lambda}\,.$$

Thus the pattern function $u(\theta) = u(\theta;x_o,y_o)$ is simply expressed in terms of the pattern functions g and n. Using (9a), (9b) and (15), and using subscripts a, b, to indicate the fulfillment of conditions (2a) or (2b), we find:

$$g_b = -\frac{2i}{3}\sqrt{\frac{2}{\pi k}}\, e^{-i\pi/4} \sum_{n=o}^{\infty} J_{\frac{2n+1}{3}}(kr_o) e^{-(2n+1/6)\pi i} \cos\frac{2n+1}{3}\theta_o \cos\frac{2n+1}{3}\theta$$

$$g_a = -\frac{2i}{3}\sqrt{\frac{2}{\pi k}}\, e^{-i\pi/4} \sum_{n=1}^{\infty} J_{\frac{2n}{3}}(kr_o) e^{-n\pi i/3} \sin\frac{2n\theta_o}{3} \sin\frac{2n\theta}{3}$$

(18)

$$n_b = -\frac{i}{3}\sqrt{\frac{2}{\pi k}}\, e^{-i\pi/4} \sum_{n=o}^{\infty} \varepsilon_n J_{\frac{2n}{3}}(kr_o) e^{-n\pi i/3} \cos\frac{2n\theta_o}{3} \cos\frac{2n\theta}{3}$$

$$n_a = -\frac{2i}{3}\sqrt{\frac{2}{\pi k}}\, e^{-i\pi/4} \sum_{n=o} J_{\frac{2n+1}{3}}(kr_o) e^{-(2n+1/6)\pi i} \sin\frac{(2n+1)\theta_o}{3} \sin\frac{2n+1}{3}\theta.$$

Equation (17) suggests a simpler determination of A if $0 < \lambda/ik \leq 1$. In this case we can write $\lambda = ik\cos\delta$ where δ is a real angle and $\cos\delta > 0$. Then, since $u(\theta;x_o,y_o)$ must be regular in the region $\frac{3\pi}{2} \geq \theta \geq 0$, it follows from (17) that

$$(19)\qquad \left[(A-1)\frac{\partial}{\partial x_o} - \lambda\right] g(\delta;x_o,y_o) + Aik\cos\delta\, n(\delta;x_o,y_o) = 0\ ,$$

so that

$$(20)\qquad A = \frac{+\left(\frac{\partial}{\partial x_o} + \lambda\right) g(\delta;x_o,y_o)}{\frac{\partial g(\delta;x_o y_o)}{\partial x_o} + ik\cos\delta\, n(\delta;x_o,y_o)}\ .$$

This formula for A is more convenient to use than the value obtainable from (14), since the latter equations involve quadratures.

Actually this procedure may be applied even when δ is not a real angle. Without entering into details in the present work, we would like to

remark that the procedure can then be based on the fact that the far field amplitude u has a half plane of regularity as a function of the complex variable k cos θ. The latter property of the function u is a consequence of the fact that the function u is the Fourier transform of the function U, which vanishes on the positive x axis (or U_y does) in view of the boundary condition.

5. Conclusion

It seems worthwhile to discuss the further applications of the method here adopted, and its relation to the methods employed by Stoker [10] in water wave theory. We have already explained, in the introduction, some of the distinctive characteristics of the water wave problem. In that problem, a wave impinges on a sloping beach or a dock. The angular space is consequently restricted to be less than or equal to π. (Stoker's dock problem corresponds to an angular space of π/2). Stoker and Lewy have determined operators L_β, and special angles $-\pi < \beta < 0$, such that the functions $L_\beta U$ obey simple boundary conditions at $\theta = 0$, $\theta = \beta$, provided U is a solution of the equation, $\triangle U - k^2 U = 0$, or $\triangle U = 0$, in the angular region $\beta < \theta < 0$, and provided U obeys mixed boundary conditions at $\theta = 0$ and $\theta = \beta$.* In addition, explicit expressions for the functions $L_\beta U$ are obtainable, by reflection in the sides of the wedge, for these particular values of β. Now inspection shows that the operator L_β, which was originally devised for the purposes of the water wave problem, is composed of derivatives with respect to x and y, and consequently depends only on the angle between those infinite lines of which the semi-infinite faces of the wedge in the water wave problem are parts. Therefore, the same operator will simplify the boundary-value problem for three other angular

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* The operator can be applied without trouble also in the case of the equation $\triangle U + k^2 U = 0$.

spaces besides the angular space envisaged in the water wave theory, so that the operators L_β can be employed in the problem of diffraction in angular spaces larger than π. For example, the operator suitable for the domain $-\frac{\pi}{4} < \theta < 0$, is also suitable for the domains $0 < \theta < \frac{7\pi}{4}$, $0 < \theta < \frac{3\pi}{4}$, and $-\frac{\pi}{4} < \theta < \pi$. However, the idea of the completion of the solution by reflection methods cannot be extended to the new domains. The more general method of separation of variables can be used instead. Let us illustrate these remarks. In the case of boundaries formed by two lines at right angles, the above consideration shows that we can simplify the boundary-value problem in the space $0 < \theta < 3\pi/2$, as well as in the space $0 > \theta > -\pi/2$. The latter space corresponds to the dock problem in the water wave investigations quoted above. The half line $\theta = -\pi/2$ then represents the water surface. For the space $0 > \theta > -\pi/2$, the type of problem occuring in diffraction theory (i.e., obeying (5)) is uninteresting mathematically. The solution can, in fact,be continued across the ray $\theta = 0$, so as to reduce to a problem of determining the Green's function for the space $-\pi/2 < \theta < \pi/2$, i.e., for $x > 0$, with the boundary condition $U_x = \lambda U$ on $x = 0$ for all y. That problem can be solved trivially by Fourier integral methods. The problem of a plane wave incident from the right becomes even more trivial, the solution being

$$U = e^{-ikx} + \frac{ik+\lambda}{ik-\lambda} e^{ikx} .$$

Of course, the type of problem posed for this space in the water wave theory is non-trivial.

As another example of a problem to which the method can be applied, consider the problem of an angular space exterior to a wedge of 45°, or 135°. The operator to be used is a simplified version of the operator employed by Isaacson [14] for the water wave problem for the space $-135^\circ < \theta < 0$.

In fact, if we let

$$\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} - \lambda \right) \left(\frac{\partial}{\partial y} + \frac{\partial}{\partial x} + \lambda \right) U = V,$$

and if we require U regular for $\pi/4 < \theta < 2\pi$, and $\partial U/\partial n = \lambda U$ on the half line $\theta = \pi/4$, and $U = 0$ on the half-line $\theta = 2\pi$, we deduce that $V = 0$ on both these half-lines. The determination of V can then proceed along the lines employed in the present paper.

It is worth noting that the method we have used can also be employed when there are mixed boundary conditions on both faces of the wedge. We have considered the simpler problem in this paper only to keep the exposition as clear as possible.

References

[1] Sommerfeld, A. - Mathematical theory of diffraction; Math. Annalen, 47, 317 (1896).

[2] MacDonald,H. - A class of diffraction problems; Proc. London Math. Soc., 14, 410 (1915).

[3] Reiche,F. - The diffraction of light by a plane right angled wedge of infinite conductivity; Ann. der Physik, 37, 131 (1912).

[4] Oberhettinger,F. - Diffraction of waves by a wedge; Comm. Pure Appl. Math., 7 , 551 (1954).

[5] Felsen, L.B. - Field solution for a class of corrugated wedge and cone surfaces; Polytech. Inst. of Brooklyn, Microwave Res. Inst., Tech. Memo. #32, July, 1957.

[6] Stoker J.J. - Surface waves in water of variable depth; Quart. Appl. Math., 5,1(1947).

[7] Lewy, H. - Waves on sloping beaches; Bull. Amer. Math. Soc., 52, 737 (1946).

[8] Weinstein, A. - On surface waves; Canad. Jrnl. of Math., 1, 271 (1949).

[9] Peters, A.S. - Water waves over sloping beaches and the solution of a mixed boundary value problem for $\triangle^2\phi - k^2\phi = 0$ in a sector; Comm. Pure Appl. Math., 5, 87 (1952).

[10] Roseau, M. - Sur les mouvements ondulatoires de la mer sur une plage; Comptes Rendus, 232, 211 (1951). Contribution a la theorie des ondes liquids de gravite en profondeur variable; Publ. Sci. et Tech. du Ministere de l'Air, No. 275, 1952.

[11] Senior, T.B.A. - Diffraction by an imperfectly conducting wedge; Univ. of Mich., Eng. Res. Inst., Studies in Radar Cross Sections, 25 (2591-2-T), Oct., 1957.

[12] Karal,F. and S.N. Karp - Diffraction of a skew plane electromagnetic wave by an absorbing right angled wedge; N.Y.U., Inst. Math. Sci., Div. EM Res., Research Report No. EM-111, Feb., 1958. Comm. on Pure and Appl. Math.; 9, No.4, Nov., 1958. (The paper there referred to as Ref.[2], is the present paper.)

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